

# The double parton distributions in the hard pomeron model

M. Braun<sup>1,a</sup>, D. Treleani<sup>2,b</sup>

<sup>1</sup> Department of High-Energy Physics, University of St. Petersburg, 198904 St.Petersburg, Russia

<sup>2</sup> Department of Theoretical Physics, University of Trieste, Strada Costiera 11, Istituto Nazionale di Fisica Nucleare, Sezione di Trieste, and ICTP, 34014, Trieste, Italy

Received: 9 May 2000 / Revised version: 14 August 2000 /  
Published online: 23 January 2001 – © Springer-Verlag 2001

**Abstract.** We study the double parton interaction process in collisions between highly virtual  $q\bar{q}$  pairs in the BFKL regime. Explicit expressions for the double parton distributions are obtained both in the case of direct coupling of the BFKL pomerons to the  $q\bar{q}$  pair and in the case of triple pomeron interaction.

## 1 Introduction

An important feature of hadronic collisions at high energies is the growth of the hard component of the interaction, induced by the increasing flux of partons. Apart from the single hard interaction, multi-parton hard interactions begin to play an increasing role. Hard events with multi-parton interactions have in fact been predicted long ago by several authors [1]. The simplest event of this kind, the double parton scattering, has been an object of experimental search in all high energy hadron collider experiments since several years [2] and while initially the results have been sparse and not very consistent, recently CDF has reported the observation of a large number of events with double parton collisions [3].

Multiple parton collisions represent a new observable feature of hadronic interactions. Non-perturbative inputs to the corresponding cross sections are the multi-parton distributions. These are new properties of the hadron structure which become accessible through the observation of multiple parton collisions. The multiple parton distributions are related directly to the many-body parton correlations in the hadron [4]. While, as a general rule, the non-perturbative inputs are quantities to be measured and cannot be computed in perturbation theory, in the interaction of two very virtual  $q\bar{q}$  pairs, the whole process falls in the domain of perturbative QCD and, in that case, multiple parton distributions can be obtained through a direct calculation, within an approximation scheme. The aim of the present paper is to study precisely this example of high energy interaction and to work out explicitly the simplest case of multi-parton distribution.

Note that studying events with several hard interactions one can introduce the standard notions of inclusive and exclusive cross sections. The first refer to the obser-

vation of at least one (or two, ...) hard events, the second of exactly one (or two, ...) hard events. The inclusive hard cross sections are most easily accessible in the experimental study. They are also most easily calculated theoretically. In fact the inclusive cross sections are basically average quantities, since they include the multiplicity factor, which is proportional to the number of hard partonic interactions [5], so it is no wonder that the inclusive cross section may become larger than the total cross section [6]. The AGK rules [7] tell that the inclusive cross sections for single, double, triple etc. hard collisions are given by the simplest relevant diagrams, which involve one, two, three etc. hard interaction blobs. All other contributions from diagrams with more hard blobs cancel in the inclusive cross sections. In the following, applying the simple theoretical formulas for the inclusive cross sections, we shall use them as basic quantities, from which we shall extract the multi-parton distributions.

The paper consists of two parts. For the sake of completeness the kinematics of the single and double parton collision and the factorization of the non-perturbative parts of the processes are re-derived in the first part of the paper. In the second part, using the perturbative QCD approach, we compute the double parton distribution in two different limiting cases. The last section is devoted to the conclusions.

## 2 Single and double parton interactions

### 2.1 Single hard scattering

To fix our notation and to settle a common ground with the double parton collision process, we discuss first the single hard scattering case which is described by the diagram shown in Fig. 1. The hard scattering corresponding to the central blob  $a$  is supposed to be characterized by a large scale  $M$  related to the concrete nature of the

<sup>a</sup> e-mail: braun1@MB1693.spb.edu

<sup>b</sup> e-mail: daniel@trieste.infn.it

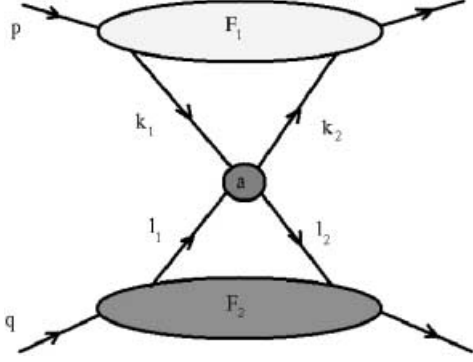


Fig. 1. Single scattering term

hard process. For processes like charm production  $M$  is obviously of the order of the mass of the charmed meson. For processes characterized by large transverse momenta of the produced particles (jet production) the scale  $M$  is determined by the minimal transverse momentum  $k_{\min}$  ascribed to a hard jet. The discontinuity of the diagram through the hard blob represents the inclusive cross section and, because of the AGK cancellation [7], it includes the factor accounting for the multiplicity of the hard partonic collisions in the hadronic interaction [5]. In the following one considers only one sort of partons which, for the hard pomeron model, is represented by the gluons. The contribution of the diagram to the inclusive cross section is related to the amplitude by

$$\sigma = (1/s)\text{Im}\mathcal{A}, \quad (1)$$

where  $s = 2p_+q_-$  is the c.m. energy squared. We neglect the masses of the projectile and target and take  $p_- = q_+ = p_\perp = q_\perp = 0$ . For massive projectile and target the standard substitution  $p \rightarrow p - \alpha q$ ,  $q \rightarrow q - \alpha p$  with  $\alpha = m^2/s$  is assumed. The usual scaling variables of the colliding partons are introduced by

$$k_+ = xp_+, \quad l_- = wq_-. \quad (2)$$

The partonic c.m. energy squared is therefore written as  $s_p \equiv M^2 = (k+l)^2 \simeq 2k_+l_- = sxw$ .

The standard treatment is based on two assumptions which are the basis for the factorization hypothesis.

(1) Parton virtualities (essentially the transverse momenta squared) are much smaller as compared with  $M^2$ .

(2) The dependence of the hard amplitude  $a$  on the virtualities of the external lines can be neglected.

The longitudinal integrations in the diagram of Fig. 1 are easily done. Indeed, according to our assumptions the amplitude  $a$  does not depend on  $k_-$ , nor on  $l_+$ . Only the upper part of the diagram depends therefore on  $k_-$ . One denotes it (with parton legs) as  $F_1(p, k)$ . Its integration over  $k_-$  is standardly transformed into an integration over the ‘‘missing mass’’  $s_1 = (p-k)^2$  of its right-hand discontinuity:

$$\begin{aligned} \int \frac{dk_-}{2\pi} F_1(p, k) &= \frac{i}{2p_+} \int_{s_{10}}^{\infty} \frac{ds_1}{\pi} \text{Im}F_1(p, k) \\ &\equiv \frac{2\pi i}{xp_+} F_1(x, k_\perp). \end{aligned} \quad (3)$$

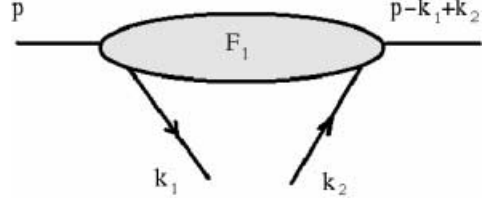


Fig. 2. Soft blob in the single scattering term

Similarly for the target

$$\begin{aligned} \int \frac{dl_+}{2\pi} F_2(q, l) &= \frac{i}{2q_-} \int_{s_{20}}^{\infty} \frac{ds_2}{\pi} \text{Im}F_2(q, l) \\ &\equiv \frac{2\pi i}{wq_-} F_2(w, l_\perp). \end{aligned} \quad (4)$$

The standard parton distributions at the scale  $M^2$  are obtained by integrating  $F_1$  and  $F_2$  over the transverse momenta  $k_\perp$  and  $l_\perp$ :

$$\rho_1(x, M^2) = \int \frac{d^2k}{(2\pi)^2} \theta(M^2 - k^2) F_1(x, k). \quad (5)$$

The final integrations over  $k_+$  and  $l_-$  can be transformed into integrations over  $x$  and  $w$ . The integration limits,  $[0, 1]$ , are a consequence of the positivity constraints of  $s_1, s_2$  and  $M^2$ . The amplitude of Fig. 1 is therefore written as

$$\mathcal{A}_1 = \int_0^1 \frac{dx dw}{xw} \rho_1(x, M^2) \rho_2(w, M^2) a(M^2), \quad (6)$$

while the single parton scattering inclusive cross sections is expressed by the usual factorized formula:

$$\sigma_1 = \int_0^1 dx dw \rho_1(x, M^2) \rho_2(w, M^2) \sigma_p(xw). \quad (7)$$

The cross section can be expressed by using the unintegrated parton distributions in coordinate space. To this end one introduces the (non-forward) target parton distribution  $F_1(x, k_{1\perp}, k_{2\perp})$ , shown in Fig. 2, and its Fourier transform

$$F_1(x, r_1, r_2) = \int \frac{d^2k_1 d^2k_2}{(2\pi)^4} F_1(x, k_1, k_2) e^{ik_1 r_1 - ik_2 r_2}. \quad (8)$$

(In the following part of this subsection one will not find 4-vectors, so we suppress the subindex  $\perp$  for the  $k$ 's.) It is convenient to introduce the partonic c.m. and relative coordinates by

$$r_1 + r_2 = 2R, \quad r_1 - r_2 = r. \quad (9)$$

One has therefore

$$\begin{aligned} F_1(x, k_1, k_2) &= \int d^2R d^2r F_1(x, R, r) e^{-iR(k_1 - k_2) - ir(k_1 + k_2)/2}, \end{aligned} \quad (10)$$

and when  $k_1$  and  $k_2$  are equal

$$F_1(x, k) = F_1(x, k, k) = \int d^2R d^2r F_1(x, R, r) e^{-ir k}, \quad (11)$$

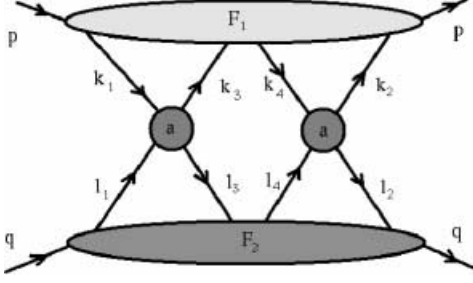


Fig. 3. Double scattering term

which is precisely the unintegrated parton distribution which appeared in the single hard scattering expression above. According to (5) one has to integrate  $F_1$  over all  $k$  values, with the condition  $k^2 \ll M^2$ , in order to obtain the final partonic distribution at the scale  $M^2$ . In the scaling approximation the  $\rho$ 's do not depend on  $M^2$ . One may therefore take the limit  $M^2 \rightarrow \infty$  and thus integrate over all  $k$  values. In the scaling approximation one therefore finds

$$\rho_1(x, M^2 \rightarrow \infty) = \int d^2R F_1(x, R, 0). \quad (12)$$

When keeping into account the limits for  $k^2$  the value  $r = 0$  is changed into  $r^2 \simeq 1/M^2$ . The standard parton distributions  $\rho(x, M^2)$ , entering in the factorization formula, can therefore be expressed via the density in coordinate space  $F$  as

$$\rho(x, M^2) = \int d^2R F(x, R, r), \quad r^2 = 1/M^2. \quad (13)$$

## 2.2 Double hard scattering

The double hard scattering diagram is shown in Fig. 3. The corresponding inclusive cross section (including the multiplicity factor) is obtained, by means of the AGK rules [7], by taking twice the imaginary part and dividing it by  $-s$ .

The previous analysis can be repeated also in this case. There are five double longitudinal integrations. As independent variables one chooses  $k_1, k_2, l_1, l_2$  and  $q = k_1 - k_3$ . The two hard scattering amplitudes are assumed to depend only on their total c.m. energies squared,  $M_1^2 = (k_1 + l_1)^2 = 2k_{1+}l_{1-}$  and  $M_2^2 = (k_2 + l_2)^2 = 2k_{2+}l_{2-}$ , not on  $k_{1(2)-}, l_{1(2)+}$ , nor on  $q_{\pm}$ . The integrations on  $k_{1(2)-}$  are made as in the single scattering case. One calls the projectile blob (with all four partonic legs)  $F_1(p, k_1, k_2, k_3, k_4)$  ( $\sum k_i = 0$ ). Then the integrations over  $k_{1(2)-}$  can be transformed into integrations over the missing masses of the two hard scatterings  $s_1 = (p - k_1)^2$  and  $s_2 = (p - k_2)^2$ , the integrand being the discontinuity of  $F$  on the corresponding right-hand cuts.

Since neither the hard scattering amplitudes nor  $F_2$  depend on  $q_-$  one may integrate  $F_1$  also over  $q_-$ . This last integration can be transformed into an integration over the missing mass in between the scatterings  $s_3 = (p - q)^2$  along

the right-hand cut, the integrand being the corresponding discontinuity. As a result one obtains

$$\begin{aligned} & \left( \frac{i}{2p_+} \right)^3 \int_{s_{10}}^{\infty} \frac{ds_1}{2\pi i} \int_{s_{20}}^{\infty} \frac{ds_2}{2\pi i} \int_{s_{30}}^{\infty} \frac{ds_3}{2\pi i} \\ & \times \text{Disc}_{s_1} \text{Disc}_{s_2} \text{Disc}_{s_3} F_1(p, k_1, k_2, k_3, k_4) \\ & \equiv -2\pi^2 i \left( \frac{1}{p_+} \right)^3 F_1(x_1, k_{1\perp}, k_{3\perp} | x_2, k_{4\perp}, k_{2\perp}). \end{aligned} \quad (14)$$

The density defined on the right-hand side is the generalization of the single to the double unintegrated partonic density. The double parton distribution of the target is introduced in an analogous way. The integrations over  $k_{1(2)+}, l_{1(2)-}$  are transformed into integrations on the scaling variables  $x_{1,2}$  and  $w_{1,2}$  and the double hard scattering amplitude of Fig. 3 is expressed as

$$\begin{aligned} \mathcal{A}_2 = & \frac{i}{2s} \int_0^1 \frac{dx_1}{x_1} \frac{dx_2}{x_2} \frac{dw_1}{w_1} \frac{dw_2}{w_2} \int \prod_{j=1}^4 \frac{d^2k_j}{(2\pi)^2} \frac{d^2l_j}{(2\pi)^2} (2\pi)^2 \\ & \delta^2(k_1 - k_2 - k_3 + k_4) \\ & \times (2\pi)^2 \delta^2(k_1 + l_1 - k_3 - l_3) \\ & \times (2\pi)^2 \delta^2(k_2 + l_2 - k_4 - l_4) \\ & F_1(x_1, k_1, k_3 | x_2, k_4, k_2) F_2(x_2, l_1, l_3 | x_2, l_4, l_2) \\ & \times a(M_1^2) a(M_2^2), \end{aligned} \quad (15)$$

where  $M_1^2 = s x_1 w_1$  and  $M_2^2 = s x_2 w_2$ . The double hard scattering cross section is therefore written as

$$\begin{aligned} \sigma_2 = & \int_0^1 dx_1 dx_2 dw_1 dw_2 \int \prod_{j=1}^4 \frac{d^2k_j}{(2\pi)^2} \frac{d^2l_j}{(2\pi)^2} \\ & (2\pi)^2 \delta^2(k_1 - k_2 - k_3 + k_4) \\ & \times (2\pi)^2 \delta^2(k_1 + l_1 - k_3 - l_3) \\ & \times (2\pi)^2 \delta^2(k_2 + l_2 - k_4 - l_4) \\ & F_1(x_1, k_1, k_3 | x_2, k_4, k_2) F_2(w_1, l_1, l_3 | w_2, l_4, l_2) \\ & \times \sigma_p(x_1 w_1) \sigma_p(x_2 w_2), \end{aligned} \quad (16)$$

and both in (15) and in (16) the integrations are done with the limits  $k_1^2, k_3^2 < M_1^2$  and  $k_2^2, k_4^2 < M_2^2$ .

The integrations over the transverse components of the transferred momenta are more conveniently performed by going to coordinates space. One introduces therefore

$$\begin{aligned} & F_1(x_1, r_1, r_3 | x_2, r_4, r_2) \\ & = \int \prod_{j=1}^4 \frac{dk_j}{(2\pi)^2} \times F_1(x_1, k_1, k_3 | x_2, k_4, k_2) \\ & \times e^{(ik_1 r_1 - ik_2 r_2 - ik_3 r_3 + ik_4 r_4)}, \end{aligned} \quad (17)$$

where in the integrand all four momenta are independent, in such a way that one is integrating also over the total momentum transferred to the projectile. One introduces the c.m. and relative coordinates defined by

$$\begin{aligned} r_1 + r_3 = 2R_1, \quad r_1 - r_3 = r_{13}, \quad r_4 + r_2 = 2R_2, \\ r_2 - r_4 = r_{24}, \end{aligned} \quad (18)$$

so that

$$\begin{aligned} r_1 &= R_1 + r_{13}/2, & r_3 &= R_1 - r_{13}/2, & r_2 &= R_2 + r_{24}/2, \\ r_4 &= R_2 - r_{24}/2. \end{aligned}$$

Similar coordinates, with primes, are introduced for the target:

$$l_1 + l_3 = 2R'_1 \quad \text{etc.}$$

Going to the coordinates space one finds, in (16), the following exponential:

$$\begin{aligned} &\exp\{-iR_1(k_1 - k_3) - ir_{13}(k_1 + k_3)/2 \\ &+ iR_2(k_2 - k_4) + ir_{24}(k_2 + k_4)/2 \\ &- iR'_1(l_1 - l_3) - ir'_{13}(l_1 + l_3)/2 \\ &+ iR'_2(l_2 - l_4) + ir'_{24}(l_2 + l_4)/2\}. \end{aligned}$$

Also the three  $\delta$  functions in (16) can be represented as integrals over three additional coordinates in transverse space:

$$\begin{aligned} (2\pi)^2 \delta^2(k_1 - k_2 - k_3 + k_4) &= \int d^2 R e^{iR(k_1 - k_2 - k_3 + k_4)}, \\ (2\pi)^2 \delta^2(k_1 + l_1 - k_3 - l_3) &= \int d^2 b_1 e^{ib_1(k_1 + l_1 - k_3 - l_3)}, \\ (2\pi)^2 \delta^2(k_4 + l_4 - k_2 - l_2) &= \int d^2 b_2 e^{ib_2(k_4 + l_4 - k_2 - l_2)}. \end{aligned}$$

In the scaling limit the integrations over the eight momenta  $k_j, l_j, j = 1, \dots, 4$  produce eight  $\delta$  functions:

$$\begin{aligned} &\delta^2(R + b_1 - R_1 - r_{13}), & \delta^2(-R - b_2 + R_2 + r_{24}), \\ &\delta^2(-R - b_1 + R_1 - r_{13}), & \delta^2(R + b_2 - R_2 + r_{24}), \\ &\delta^2(b_1 - R'_1 - r'_{13}), & \delta^2(-b_2 + R'_2 + r'_{24}), \\ &\delta^2(-b_1 + R'_1 - r'_{13}), & \delta^2(b_2 - R'_2 + r'_{24}). \end{aligned}$$

The integrations over the transverse coordinates then give  $r_{13} = r'_{13} = r_{24} = r'_{24} = 0$ ,  $R'_1 = b_1$ ,  $R'_2 = b_2$ ,  $R_1 = R + b_1$ ,  $R_2 = R + b_2$ . As a result the double hard cross section is expressed as an integral over the three impact parameters,  $R, b_1$  and  $b_2$

$$\begin{aligned} \sigma_2 &= \int_0^1 dx_1 dx_2 dw_1 dw_2 \sigma_p(x_1 w_1) \sigma_p(x_2 w_2) \\ &\int d^2 R d^2 b_1 d^2 b_2 F_1(x_1, R + b_1, 0 | x_2, R + b_2, 0) \\ &\times F_2(w_1, b_1, 0 | w_2, b_2, 0). \end{aligned} \quad (19)$$

As in the case of the single parton scattering process, to keep into account that the integrations on the transverse momenta cannot be extended to infinity, one takes the relative transverse distances between the interacting partons to be of order of the dimensions of the two hard scattering blobs, namely is  $1/M_1$  and  $1/M_2$ . The resulting expression of the double parton scattering cross section is therefore written as

$$\begin{aligned} \sigma_2 &= \int_0^1 dx_1 dx_2 dw_1 dw_2 \sigma_p(x_1 w_1) \sigma_p(x_2 w_2) \\ &\times \int d^2 R d^2 b_1 d^2 b_2 F_1(x_1, R + b_1, r_{13} | x_2, R + b_2, r_{24}) \\ &\times F_2(w_1, b_1, r_{13} | w_2, b_2, r_{24}), \end{aligned} \quad (20)$$

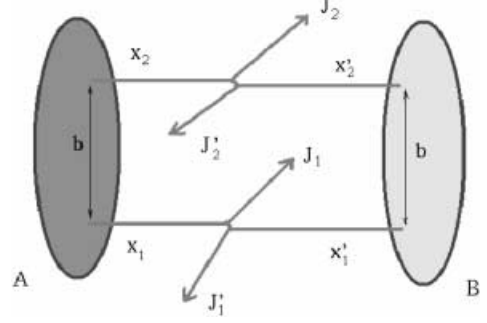


Fig. 4. Graphical representation of (22)

with  $r_{13}^2 = 1/M_1^2$  and  $r_{24}^2 = 1/M_2^2$ .

The expression is further simplified by introducing the c.m. and relative parton coordinates in transverse space

$$B_1 = (b_1 + b_2)/2 + R, \quad B_2 = (b_1 + b_2)/2, \quad b = b_1 - b_2.$$

The double parton distributions are therefore defined as

$$\begin{aligned} &\Gamma_1(x_1, x_2, b; M_1^2, M_2^2) \\ &= \int d^2 B_1 \times F_1(x_1, R + b_1, r_{13} | x_2, R + b_2, r_{24}), \\ &\Gamma_2(w_1, w_2, b; M_1^2, M_2^2) \\ &= \int d^2 B_2 \times F_2(w_1, b_1, r_{13} | w_2, b_2, r_{24}), \end{aligned} \quad (21)$$

and the double parton scattering cross section assumes a simpler form

$$\begin{aligned} \sigma_2 &= \int \Gamma_1(x_1, x_2, b; M_1^2, M_2^2) \sigma_p(x_1 w_1) \sigma_p(x_2 w_2) \\ &\times \Gamma_2(w_1, w_2, b; M_1^2, M_2^2) dx_1 dw_1 dx_2 dw_2 d^2 b, \end{aligned} \quad (22)$$

with the geometrical interpretation shown in Fig. 4. Notice that the normalization contains also the multiplicity factor which counts the number of parton interactions in the hadronic collision (two in this case).

## 3 Double parton distributions in the BFKL regime

### 3.1 Emission of a jet from a single BFKL chain

In the framework of the hard pomeron model the hard collision process is associated to the production of a gluon with sufficiently large transverse momentum  $k > k_{\min}$  and at a given rapidity (in this section only 2-dimensional transverse vectors will appear, so that we again omit the subindex  $\perp$  and one take the metric Euclidean,  $k^2 \geq 0$ ). The kinematics of the process is clear from Fig. 5.

For the forward case,  $p_1 = p'_1$ , the emission of a gluon with momentum  $k$  is described by modifying as follows the BFKL Green function in coordinates space  $G(r', r'')$  [8]:

$$G(r', r'') \rightarrow \int d^2 r G(r'r) V(k) G(r, r''), \quad (23)$$

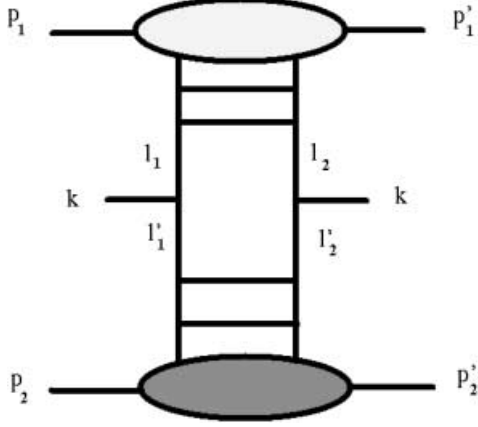


Fig. 5. Emission of a jet from a BFKL chain

where  $r$ 's are relative distances between the gluons and

$$V_k(r) = \frac{12\alpha_s}{k^2} \overleftarrow{\Delta} e^{ikr} \overrightarrow{\Delta}, \quad (24)$$

is the vertex operator describing the emission of the gluon; the arrows show the direction of the action of the  $\Delta$  operators. The two BFKL Green functions in (23) are to be taken at appropriate energy (rapidity) ranges, corresponding to the rapidity distance of the emitted gluon from the projectile. This dependence is not written explicitly to avoid overloading our formulas with arguments and/or indices.

The first step is to generalize (23) for the non-forward BFKL chain. In momentum space the emission of a gluon with momentum  $k$  is described by the vertex

$$\begin{aligned} V(l_1, l_2 | l'_1, l'_2) &= \frac{6\alpha_s}{k^2} (2\pi)^2 \delta^2 \left( \frac{1}{2}(l_1 + l_2 - l'_1 - l'_2) - k \right) \\ &\times (l_1^2 l_2'^2 + l_2^2 l_1'^2 - k^2 (l_1 - l_2)^2). \end{aligned} \quad (25)$$

So the emission blob in momentum space  $Z_k(q_1, q_2 | q'_1, q'_2)$  is given by

$$\begin{aligned} Z_k(q_1, q_2 | q'_1, q'_2) &= \frac{6\alpha_s}{k^2} \int \prod_{i=1}^2 \frac{d^2 l_i}{(2\pi)^2} \frac{d^2 l'_i}{(2\pi)^2} (2\pi)^2 \\ &\times \delta^2 \left( \frac{1}{2}(l_1 + l_2 - l'_1 - l'_2) - k \right) \\ &\times (l_1^2 l_2'^2 + l_2^2 l_1'^2 - k^2 (l_1 - l_2)^2) \\ &\times G(q_1, q_2 | l_1, l_2) G(l'_1, l'_2 | q'_1, q'_2) \\ &\times (2\pi)^2 \delta^2(q_1 - q_2 - l_1 + l_2) \\ &\times (2\pi)^2 \delta^2(l'_1 - l'_2 - q'_1 + q'_2). \end{aligned} \quad (26)$$

In coordinates space one writes

$$\begin{aligned} (2\pi)^2 \delta^2(q_1 - q_2 - l_1 + l_2) G(q_1, q_2 | l_1, l_2) &= \int \prod_{i=1}^2 d^2 r_i d^2 z_i G(r_1, r_2 | z_1, z_2) \\ &\times e^{(-iq_1 r_1 + iq_2 r_2 + il_1 z_1 - il_2 z_2)}, \end{aligned} \quad (27)$$

and

$$\begin{aligned} (2\pi)^2 \delta^2(l'_1 - l'_2 - q'_1 + q'_2) G(l'_1, l'_2 | q'_1, q'_2) &= \int \prod_{i=1}^2 d^2 r'_i d^2 z'_i G(z'_1, z'_2 | r'_1, r'_2) \\ &\times e^{(-il'_1 z'_1 + il'_2 z'_2 + iq'_1 r'_1 - iq'_2 r'_2)}. \end{aligned} \quad (28)$$

Note that due to the inclusion of the  $\delta$  function in the Fourier transforms (25) and (26), the Green functions in coordinate space are translationally invariant:

$$G(r_1 + a, r_2 + a | z_1 + a, z_2 + a) = G(r_1, r_2 | z_1, z_2). \quad (29)$$

The momenta squared in the emission vertex are substituted by differential operators applied to the Green function. The bracket in (24) is written as

$$\Delta_1 \Delta'_2 + \Delta_2 \Delta'_1 + k^2 (\nabla_1 + \nabla_2) (\nabla'_1 + \nabla'_2),$$

where one uses the notation  $\Delta_1 = \Delta(z_1)$ ,  $\Delta'_1 = \Delta(z'_1)$  etc. The remaining  $\delta$  function in (26) is represented as an integral

$$\begin{aligned} (2\pi)^2 \delta^2((l_1 + l_2 - l'_1 - l'_2)/2 - k) &= \int d^2 z e^{-iz((l_1 + l_2 - l'_1 - l'_2)/2 - k)}, \end{aligned}$$

in such a way that the integrations over the four transverse momenta in (24) produce four  $\delta$  functions in coordinate space:

$$\begin{aligned} \delta^2(z_1 - z/2), \quad \delta^2(z_2 + z/2), \quad \delta^2(z'_1 - z/2), \\ \delta^2(z'_2 + z/2). \end{aligned}$$

One obtains therefore

$$\begin{aligned} Z_k(q_1, q_2 | q'_1, q'_2) &= \int \prod_{i=1}^2 d^2 r_i d^2 r'_i e^{(-iq_1 r_1 + iq_2 r_2 + iq'_1 r'_1 - iq'_2 r'_2)} d^2 z \\ &\times \left[ G(r_1, r_2 | z_1, z_2) \frac{6\alpha_s}{k^2} e^{ikz} \right. \\ &\times (\Delta_1 \Delta'_2 + \Delta_2 \Delta'_1 + k^2 (\nabla_1 + \nabla_2) (\nabla'_1 + \nabla'_2)) \\ &\left. \times G(z'_1, z'_2 | r'_1, r'_2) \right]_{z_1=z'_1=z/2, z_2=z'_2=-z/2}, \end{aligned} \quad (30)$$

and the differential operator which stands between the two Green functions is precisely the generalization of the emission operator  $V_k$  to the non-forward case. By using the notation

$$\begin{aligned} G \left( r_1, r_2 \middle| \frac{z}{2}, -\frac{z}{2} \right) V_k(z) G \left( \frac{z}{2}, -\frac{z}{2} \middle| r'_1, r'_2 \right) &\equiv \left[ G(r_1, r_2 | z_1, z_2) \frac{6\alpha_s}{k^2} e^{ikz} \right. \\ &\times (\Delta_1 \Delta'_2 + \Delta_2 \Delta'_1 + k^2 (\nabla_1 + \nabla_2) (\nabla'_1 + \nabla'_2)) \\ &\left. \times G(z'_1, z'_2 | r'_1, r'_2) \right]_{z_1=z'_1=z/2, z_2=z'_2=-z/2}, \end{aligned} \quad (31)$$

one may rewrite (28) in the more compact form

$$\begin{aligned} & Z_k(q_1, q_2 | q'_1, q'_2) \\ &= \int \prod_{i=1}^2 d^2 r_i d^2 r'_i e^{(-iq_1 r_1 + iq_2 r_2 + iq'_1 r'_1 - iq'_2 r'_2)} d^2 z \\ &\times G\left(r_1, r_2 \mid \frac{z}{2}, -\frac{z}{2}\right) V_k(z) G\left(\frac{z}{2}, -\frac{z}{2} \mid r'_1, r'_2\right). \end{aligned} \quad (32)$$

One defines  $Z$  in coordinates space in the following way:

$$\begin{aligned} & Z_k(r_1, r_2 | r'_1, r'_2) \\ &= \int \prod_{j=1}^2 \frac{d^2 q_j}{(2\pi)^2} \frac{d^2 q'_j}{(2\pi)^2} (2\pi)^2 \delta(q_1 - q_2 - q'_1 + q'_2) \\ &\times Z_k(q_1, q_2 | q'_1, q'_2) e^{(iq_1 r_1 - iq_2 r_2 - iq'_1 r'_1 + iq'_2 r'_2)}. \end{aligned} \quad (33)$$

By expressing the  $\delta$  function as an integral over the impact parameters  $B$

$$(2\pi)^2 \delta(q_1 - q_2 - q'_1 + q'_2) = \int d^2 B e^{-iB(q_1 - q_2 - q'_1 + q'_2)}, \quad (34)$$

and by using (32) and (34) in (33) one can do all integrations with the exception of those on  $B$  and  $z$ . The result is

$$\begin{aligned} & Z_k(r_1, r_2 | r'_1, r'_2) \\ &= \int d^2 B d^2 z G\left(r_1 + B, r_2 + B \mid \frac{z}{2}, -\frac{z}{2}\right) \\ &\times V_k(z) G\left(\frac{z}{2}, -\frac{z}{2} \mid r'_1 + B, r'_2 + B\right), \end{aligned} \quad (35)$$

and, as a function of the c.m. and relative coordinates,

$$\begin{aligned} & Z_k(R, r | R', r') = \int d^2 B d^2 z G(R + B, r | 0, z) \\ &\times V_k(z) G(0, z | R' + B, r'), \end{aligned} \quad (36)$$

showing that  $Z_k$  is translationally invariant:

$$\begin{aligned} & Z_k(R, r | R', r') = Z_k(R + B, r | R' + B, r') \\ &= Z_k(R - R', r, r'). \end{aligned} \quad (37)$$

Equation (37) is our final expression. It shows how one should change a BFKL Green function  $G(R, r | R', r') = G(R - R', r, r')$  to describe the emission of a gluon from the BFKL chain.

### 3.2 Single parton distributions

The simplest case to consider in the BFKL formalism is the inclusive emission of a gluon. The problem has been extensively studied in the literature [9]. Here we approach the problem from a somewhat novel point of view, introducing the single parton densities in coordinates space in the BFKL framework. The quantities which are factorized in the BFKL formalism are the ‘‘unintegrated gluon densities’’ [10], depending explicitly on the transverse momentum of the interacting gluon. The distributions that will

be discussed hereafter are therefore different as compared to the distributions discussed in the first part of the paper, since they depend explicitly on the distance in transverse space between the initial and the final interacting partons. The latter distributions are therefore obtained only after integration on this transverse distance, with  $1/k$  as a lower limit.

Rather than using the known expression for the inclusive cross section in terms of forward BFKL Green functions, one starts from the amplitude, corresponding to the exchange of any number of pomerons between the projectile and target, obtained in the approximation of a large number of colors  $N$  and assuming the coupling of pomerons to the colliding particles as purely perturbative [11]. The amplitude is written as

$$\begin{aligned} & A = 2is \int d^2 R \int d^2 r \int d^2 r' \rho_p(r) \rho_q(r') \\ &\times \left(1 - e^{-\frac{1}{2} g^4 G_s(R, r, r')}\right). \end{aligned} \quad (38)$$

Here  $\rho_{p(q)}(r)$  are the color densities of the projectile and of the target (with momenta  $p$  and  $q$  respectively). The function  $G_s(R, r, r') = G(R, r | 0, r')$  is the BFKL Green function, depending on the Mandelstam invariant  $s$ . When retaining the  $s$ -wave only, because of the azimuthal symmetry, it takes the form [12]

$$\begin{aligned} & G_s(R, r, r') = \frac{1}{(2\pi)^4} \int d\nu \frac{\nu^2 s^{-E(\nu)}}{(\nu^2 + 1/4)^2} \int d^2 r_0 \\ &\times \left(\frac{r}{r_{10} r_{20}}\right)^{1+2i\nu} \left(\frac{r'}{r'_{10} r'_{20}}\right)^{1-2i\nu}, \end{aligned} \quad (39)$$

where

$$\begin{aligned} & r_{10} = R + r/2 - r_0, \quad r_{20} = R - r/2 - r_0, \\ & r'_{10} = r'/2 - r_0, \quad r'_{20} = -r'/2 - r_0, \end{aligned}$$

and  $E(\nu)$  is the BFKL pomeron energy

$$E(\nu) = (3g^2/2\pi^2)(\text{Re}\psi(1/2 + i\nu) - \psi(1)). \quad (40)$$

Equation (38) allows us to easily write down the amplitudes for single, double etc. pomeron exchange processes. The single BFKL pomeron exchange amplitude is written as

$$A_1 = is g^4 \alpha_s^2 \int d^2 R \int d^2 r \int d^2 r' \rho_p(r) \rho_q(r') G_s(R, r, r'), \quad (41)$$

where the factor  $g^4$  corresponds to the couplings of the pomeron to the color densities  $\rho$ . The inclusive cross section for emission of a single gluon therefore has the form

$$\begin{aligned} & I(k) = \frac{(2\pi)^2 d^3 \sigma}{dy d^2 k} \\ &= g^4 \int d^2 R \int d^2 r \int d^2 r' \rho_p(r) \rho_q(r') Z_k(R, r, r') \\ &= \int d^2 R \int d^2 r \int d^2 r' g^2 \rho_p(r) g^2 \rho_q(r') d^2 B d^2 z \end{aligned}$$

$$\begin{aligned}
& \times G_{s_1}(R+B, r | 0, z) V_k(z) G_{s_2}(0, z | B, r') \\
& = \int d^2 R d^2 R' d^2 z \int d^2 r \int d^2 r' g^2 \rho_p(r) g^2 \rho_q(r') \\
& \times G_{s_1}(R, r | 0, z) V_k(z) G_{s_2}(0, z | R', r'). \quad (42)
\end{aligned}$$

The dependence on energy is written explicitly in the Green functions. The emitted gluon rapidity (relative to the target) is  $y = \log s_2$ . Then  $s_1 = s/s_2$  (the scale is not defined in the theory; as usual one sets it to 1 GeV). The integrated single hard scattering inclusive cross section can be written as

$$\sigma_1 = \int dy d^2 k I(k) = \int d^2 k \int_0^1 dx dw \delta\left(xw - \frac{k^2}{s}\right) I(k),$$

where one has introduced the scaling variables  $x = s_1/s$  and  $w = s_2/s$ . By comparing the expression above with the factorization formula (7) (with the densities  $\rho$  expressed via (13)) and associating the hard scattering cross section with the emission vertex  $V_k$  one obtains the expression for the partonic density of the projectile

$$F_p(x, R, z) = g^2 \int d^2 r \rho_p(r) G_{s_1}(R, r, z), \quad (43)$$

with  $x = s/s_1$  and  $G(R, r, z) = G(R, r | 0, z)$ .

The meaning of the variables is the following:  $R$  is the distance from the center of the projectile and  $z$  the distance between the initial and the final interacting partons. The size of  $z$  is of order of  $1/k$ , as a result of the factor  $\exp ikz$  in the hard interaction vertex, and the size of  $r$  is of the order of the dimensions of the projectile. The perturbative approach is justified when both  $r$  and  $z$  are small. The asymptotics of the Green function  $G$  for small relative distances and large energy was obtained in [11]. The resulting expression is a function of the dimensionless variable  $\xi = R^2/r r' (\gg 1)$ :

$$\begin{aligned}
G_s(R, r, r')_{r, r' \ll R} & \simeq \frac{s^\Delta}{\pi^{5/2} (a \ln s)^{3/2}} \xi^{-1} \\
& \times \ln \xi \exp\left(-\frac{\ln^2 \xi}{a \ln s}\right), \quad (44)
\end{aligned}$$

where  $\Delta$  is the BFKL intercept

$$\Delta = \frac{12\alpha_s}{\pi} \ln 2, \quad (45)$$

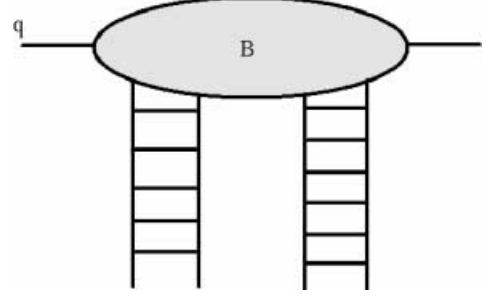
and

$$a = \frac{42\alpha_s}{\pi} \zeta(3). \quad (46)$$

With  $z$  small and fixed, the partonic density (43) behaves as

$$F_p(x, R, z) \sim \frac{(1/x)^\Delta}{\log^{3/2}(1/x)} \frac{1}{R^2} \log R \exp\left(-\frac{\log^2 R}{a \log(1/x)}\right), \quad (47)$$

and it falls rather slowly (essentially as  $1/R^2$ ) as a function of the distance from the center of the projectile ( $R$



**Fig. 6.** Double parton density in the case of direct coupling of the BFKL pomerons to the constituent quarks

represents the distance from the projectile, whose average dimension  $r_0 = \langle r \rangle$  is much smaller than  $R$ ). The effect of the exponential is felt only at relatively large  $R \sim \exp(1/x)^{1/2}$ . A more quantitative estimate of the density can be made by neglecting the weak dependence on  $r$  in the logarithmic factors in (44) and by making the substitution  $r \rightarrow r_0$ :

$$\begin{aligned}
F_p(x, R, z) & \simeq \frac{g^2}{\pi^{5/2}} \frac{(1/x)^\Delta}{(a \ln(1/x))^{3/2}} \frac{z r_0}{R^2} \ln \frac{R^2}{z r_0} \\
& \times \exp\left(-\frac{\ln^2(R^2/z r_0)}{a \ln(1/x)}\right). \quad (48)
\end{aligned}$$

An analogous expression can be written for the target.

### 3.3 Two pomerons directly coupled to the projectile (target) and triple pomeron

The double partonic density enters in the process of producing two gluons, which physically corresponds to two jets. It corresponds to the emission of the two gluons from two different BFKL chains. Note that two gluons can also be produced from a single BFKL chain. This process is however subdominant in energy in the BFKL model, being damped by a power of  $s$ . The general structure of the amplitude describing the double partonic density in the BFKL regime is shown in Fig. 6. The coupling of the two pomerons to the projectile is described by the upper blob  $B$ . One calls the energetic variable of the pomeron  $s_2 = s/s_1$ , with  $s$  the overall c.m. energy squared; for simplicity one takes it to be the same for both legs. The upper blob  $B$  is then integrated on  $s_1$ .

The asymptotic behavior of  $B(s_1)$  is described by a single pomeron exchange, so that

$$B(s_1) \equiv B(y_1) = b(y_1) e^{\Delta y_1}, \quad y_1 = \log s_1, \quad (49)$$

where  $\Delta$  is the pomeron intercept and  $b(y)$  is some smooth function tending to a constant at high  $y_1$ . Each of the pomeron legs has a similar behavior as a function of its energy variable  $s_2$ :

$$P(s_2) \equiv P(y_2) = p(y_2) e^{\Delta y_2}, \quad y_2 = \log s_2, \quad (50)$$

with  $p(y_2)$  a smooth function tending to a constant at high  $y_2$ . The amplitude of Fig. 6 is given by an integral over  $y_1$ :

$$\begin{aligned} & \int_0^Y dy_1 B(y_1) P_1(Y - y_1) P_2(Y - y_1) \\ &= e^{2\Delta Y} \int_0^Y dy_1 e^{-\Delta y_1} b(y_1) p_1(Y - y_1) p_2(Y - y_1), \end{aligned} \quad (51)$$

where  $Y = y_1 + y_2 = \log s$  is the overall rapidity interval. As one may see, at very large  $Y$  values, the integration in (51) stays limited to the region where  $y_1$  is smaller than or of the order  $1/\Delta$  and thus it is independent on  $Y$ . One may therefore take for  $p_{1(2)}$  the asymptotic values and rewrite (51) as

$$e^{2\Delta Y} p_1^{\text{as}} p_2^{\text{as}} \int_0^\infty e^{-\Delta y_1} b(y_1) dy_1, \quad (52)$$

which corresponds precisely to the behavior of a two pomeron exchange process.

The rapidities that dominate the integral (52) are of order  $1/\Delta$ . The actual value  $\Delta$  is, however, not well defined, since it is given by the coupling, whose value is a parameter in the BFKL approach. Rather than trying to determine which are the rapidities relevant to  $B$ , we proceed by discussing two limiting cases, which admit perturbative treatment.

Let the projectile be a virtual photon (or a heavy meson, ‘‘onium’’) characterized by a large virtuality  $Q^2$  (large onium mass). Then a limiting configuration is reached when  $Q^2$  is of the same order as (or larger than)  $s_1$ , namely

$$\log Q^2 \sim (\text{or } >) 1/\Delta \ll Y. \quad (53)$$

The blob  $B$  enters then in the DGLAP regime (finite scaling variable  $x_1$ ) where only the large logarithms  $\log(Q^2/\Lambda)$  are to be considered. In the fixed coupling approach of BFKL these logarithms are neglected altogether. Within the present scale invariant model one does not need therefore to sum any logarithms and one may stay within the pure perturbative approach. The upper blob is then simply reduced to the  $q\bar{q}$  pair, into which the photon (onium) has decayed and to which the pomerons are coupled directly. This is precisely the approximation which leads to the amplitude (36) obtained in [11] and which was used in the previous subsection.

On the other hand, at lower virtualities, such as

$$1 \ll \log Q^2 \ll 1/\Delta, \quad (54)$$

the structure of the blob  $B$  becomes important. Another limiting configuration is reached when  $B$  enters in the BFKL regime, namely it is itself described by a pomeron, which eventually has to split into the two lower pomerons and the whole amplitude is described by a triple pomeron interaction. One should keep in mind that, in this last case, the three pomerons are not, however, in an equivalent regime. The two lower ones, as mentioned, may be taken at their asymptotical regime,  $\Delta y_2 \gg 1$ . The upper one, on the contrary, is characterized by much smaller energies, such that  $\Delta y_1 \sim 1$ , so that its exact form needs to be used.

The two limiting possibilities which we consider are therefore the direct coupling of the pomerons to the constituent quarks and the triple pomeron interaction. We first discuss the simpler direct coupling case.

### 3.4 Two pomerons coupled directly to the projectile (target)

From the general expression of the amplitude (36) the contribution with two pomerons directly coupled both to the projectile and target is written as

$$A_2 = -is \frac{g^8}{2} \int d^2 R \int d^2 r \int d^2 r' \rho_p(r) \rho_q(r') G_s^2(R, r, r'). \quad (55)$$

The integrated inclusive cross section is obtained by dividing the amplitude by  $s$  and by taking the imaginary part with a minus sign:

$$\sigma_2 = g^8 \int d^2 R \int d^2 r \int d^2 r' \rho_p(r) \rho_q(r') G_s^2(R, r, r'). \quad (56)$$

To obtain the double differential inclusive cross section, corresponding to the emission of two hard gluons with momenta  $k_1$  and  $k_2$  from the two pomerons, we have to substitute the pomeronic Green function with the emission functions  $Z_{k_1}$  and  $Z_{k_2}$ . One obtains therefore

$$\begin{aligned} I(k_1, k_2) &= \frac{(2\pi)^2 d^3}{dy_1 d^2 k_1} \frac{(2\pi)^2 d^3}{dy_2 d^2 k_2} \sigma \\ &= g^8 \int d^2 R \int d^2 r \int d^2 r' \rho_p(r) \rho_q(r') \\ &\quad \times Z_{k_1}(R, r, r') Z_{k_2}(R, r, r'). \end{aligned} \quad (57)$$

Explicitly, in terms of pomeron Green functions and emission vertices, one has

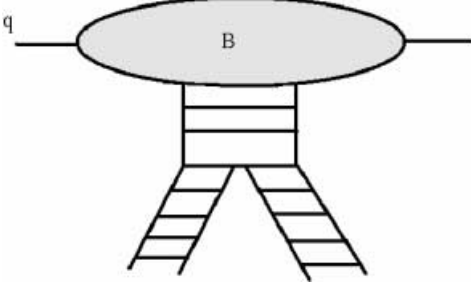
$$\begin{aligned} I(k_1, k_2) &= \int d^2 R \int d^2 r \int d^2 r' \rho_p(r) \rho_q(r') \\ &\quad \times \int d^2 R_1 d^2 R_2 d^2 z_1 d^2 z_2 \\ &\quad \times G_{s_1}(R + R_1, r | 0, z_1) V_{k_1}(z_1) \\ &\quad \times G_{s/s_1}(0, z_1 | R_1, r') \\ &\quad \times G_{s_2}(R + R_2, r | 0, z_2) V_{k_2}(z_2) \\ &\quad \times G_{s/s_2}(0, z_2 | R_2, r'). \end{aligned} \quad (58)$$

By comparing this expression with the standard form of the double hard scattering cross section one may identify the double partonic density of the projectile

$$\begin{aligned} F_p(x_1, R_1, z_1 | x_2, R_2, z_2) \\ = g^4 \int d^2 r \rho_p(r) G_{s_1}(R_1, r | 0, z_1) G_{s_2}(R_2, r | 0, z_2), \end{aligned} \quad (59)$$

where  $\log s_{1(2)} = -\log x_{1(2)}$ . A similar formula holds for the target. The density (59) can be further integrated over  $1/2(R_1 + R_2)$  to obtain the double parton distribution depending only on the distance between the partons  $\Gamma_p(x_1, x_2, R_1 - R_2, z_1, z_2)$  defined by (21).





**Fig. 7.** Double parton density in the triple pomeron case

The expression (59) is a direct generalization of the single parton density (43) and it can obviously be generalized to the multiple partonic densities generated by the direct coupling of any number of pomerons to the projectile (target)

$$F(x_1, R_1, z_1 | \cdots | x_n, R_n, z_n) = g^{2n} \int d^2r \rho_p(r) \prod_{i=1}^n G_{s_i}(R_i, r | 0, z_i)$$

where  $\log x_i = -\log s_i$ . (60)

In this simple picture the multiple partonic distributions are factorized under the integral over  $r$ , which labels the different configurations of the  $q\bar{q}$  pair. Interestingly this is precisely the correlation in transverse space which has recently been suggested [13] to describe the anomalously small value of the effective cross section measured by the CDF experiment [3].

An estimate of the many-body parton distribution can be made, in this case, by using the asymptotics (42) and by substituting  $r$  in all slowly varying (logarithmic) factors by its (small) average value  $\langle r \rangle$ . One obtains

$$F_p(x_1, R_1, z_1 | \cdots | x_n, R_n, z_n) \simeq \frac{\langle r^n \rangle}{\langle r \rangle^n} \prod_{i=1}^n F_p(x_i, R_i, z_i),$$

(61)

where  $F_p(x, R, z)$  is the single parton distribution defined by (41). Although the multi-parton distributions are just a product of single ones, they contain the factor  $\langle r^n \rangle / \langle r \rangle^n$  which is always greater than unity. The consequence is a positive correlations between partons, which however is independent of their position in transverse space. As an example the factor  $\langle r^2 \rangle / \langle r \rangle^2$  is equal to  $4/\pi$  for the Gaussian distribution  $\rho(r)$  and  $3/2$  for the exponential one.

### 3.5 Triple pomeron: generalities

In Fig. 7 we show the amplitude in the case of a triple pomeron interaction which, in the large number of colors  $N$  limit, has been discussed in [14]. The resulting expression is written as

$$\mathcal{A}_{\text{TP}}(s_1) = -is_1 \frac{g^4 N}{2\pi^3} \int_0^{y_1} dy \int \frac{d^2r_1 d^2r_2 d^2r_3}{r_{13}^2 r_{12}^2 r_{23}^2}$$

$$\times \Psi_1(r_1, r_2; y_1 - y) \Psi_2(r_2, r_3; y_1 - y) \times r_{13}^4 \nabla_1^2 \nabla_3^2 \Psi(r_1, r_3; y). \quad (62)$$

Here and in the future we use the notations  $r_{12} = r_1 - r_2$ ,  $R_{12} = (r_1 + r_2)/2$  etc. The  $y$ 's are the corresponding rapidity intervals. The pomeronic amplitudes  $\Psi$  are defined as follows:

$$\Psi(r_1, r_3, y) = \frac{1}{2} g^2 \int d^2R'_{13} d^2r'_{13} \rho(r'_{13}) G_{s_0}(r'_1, r'_3 | r_1, r_3), \quad (63)$$

with  $s_0 = e^y$  and similarly for the other two. To obtain the corresponding double partonic density, one has to substitute (1) the factor  $-is_1$  by 2, and (2) the two pomeron amplitudes  $\Psi_1$  and  $\Psi_2$  by the appropriate pomeronic Green functions with free legs describing the gluons taking part in the hard interaction. The functions may have different rapidity intervals, corresponding to the different rapidities  $y_1$  and  $y_2$  of the interacting gluons. The amplitude (62) is then converted to

$$\frac{g^6 N}{8\pi^3} \int_0^{y_{\min}} dy \int d^2R'_{13} d^2r'_{13} \rho(r'_{13}) \int \frac{d^2r_1 d^2r_2 d^2r_3}{r_{13}^2 r_{12}^2 r_{23}^2} \times G_{s_1/s_0}(r_1, r_2 | z_1, z_3) G_{s_2/s_0}(r_2, r_3 | z_2, z_4) \times r_{13}^4 \nabla_1^2 \nabla_3^2 G_{s_0}(r'_1, r'_3 | r_1, r_3), \quad (64)$$

where  $y = \log s_0$ ,  $y_{\min} = \min\{y_1, y_2\}$ .

Before moving further, one has to eliminate the differential operator acting on the last Green function, which makes the incoming pomeron asymmetric with respect to the two outgoing ones in the triple vertex. To this end it is sufficient to note that the differential operator is proportional to the Casimir operator of the conformal group

$$r_{13}^4 \nabla_1^2 \nabla_2^2 = 16C_2.$$

Acting on the function

$$\left( \frac{r_{13}}{r_{10} r_{30}} \right)^{1+2i\nu}$$

which appears in  $G(r'_1, r'_3 | r_1, r_3)$  it gives a factor  $16(\nu^2 + 1/4)^2$ . The action of the operator on  $G$  is therefore to transform it into a Green function which lacks the denominator in (39):

$$\tilde{G}_s(R, r, r') = \frac{1}{\pi^4} \int d\nu \nu^2 s^{-E(\nu)} \int d^2r_0 \times \left( \frac{r}{r_{10} r_{20}} \right)^{1+2i\nu} \left( \frac{r'}{r'_{10} r'_{20}} \right)^{1-2i\nu}. \quad (65)$$

Note that this function has the same asymptotics as  $G$  at high  $s$ , since it is determined by values of  $\nu$  close to zero.

To obtain the double partonic density one has to finally take  $Z_{13} = Z_{24} = 0$ , and to fix the c.m. coordinates of the initial gluons in both pomeronic legs  $R_{12} \equiv R_1$  and  $R_{23} \equiv R_2$ . Note that the triple vertex coordinates  $r_i$ ,  $i = 1, 2, 3$

can be expressed via the c.m. coordinates of the initial gluons

$$\begin{aligned} r_1 &= R_{12} + R_{31} - R_{23}, & r_2 &= R_{12} + R_{23} - R_{13}, \\ r_3 &= R_{23} + R_{13} - R_{12}, \end{aligned} \quad (66)$$

so that

$$\begin{aligned} r_{12} &= 2(R_{13} - R_{23}), & r_{23} &= 2(R_{12} - R_{13}), \\ r_{31} &= 2(R_{23} - R_{12}). \end{aligned} \quad (67)$$

The jacobian of the transformation from  $r_1, r_2, r_3$  to  $R_{12}, R_{23}, R_{31}$  is 4. To obtain the density one has to drop the integrations over  $R_{12}$  and  $R_{23}$ . The double parton density in the triple pomeron interaction case is therefore

$$\begin{aligned} F(x_1, R_1, z_1 | x_2, R_2, z_2) &= \frac{g^6 N}{128\pi^3} \frac{1}{(R_1 - R_2)^2} \\ &\times \int_0^{y_{\min}} dy \\ &\int d^2 r' \rho(r') \int \frac{d^2 R' d^2 R}{(R - R_1)^2 (R - R_2)^2} \\ &\times G_{s_1/s_0}(R_1, 2(R_2 - R), z_1) G_{s_2/s_0}(R_2, 2(R_1 - R), z_2) \\ &\times \tilde{G}_{s_0}(R' - R, r', 2(R_1 - R_2)). \end{aligned} \quad (68)$$

One can decouple the space integration involving the upper pomeron by shifting the integration variable  $R' \rightarrow R' - R$ . The expression is then written as

$$\begin{aligned} F(x_1, R_1, z_1 | x_2, R_2, z_2) &= \frac{g^6 N}{128\pi^3} \frac{1}{(R_1 - R_2)^2} \\ &\times \int_0^{y_{\min}} dy \int d^2 R' d^2 r' \rho(r') \tilde{G}_{s_0}(R', r', 2(R_1 - R_2)) \\ &\times \int \frac{d^2 R}{(R - R_1)^2 (R - R_2)^2} G_{s_1/s_0}(R_1, 2(R_2 - R), z_1) \\ &\times G_{s_2/s_0}(R_2, 2(R_1 - R), z_2). \end{aligned} \quad (69)$$

### 3.6 Triple pomeron: calculation

To evaluate the expression (69) one needs to make some simplifications: We use the asymptotic expressions for the two lower pomerons and we keep explicitly into account that the two relative distances  $z_{1,2}$  are much smaller as compared to the other distance in (69), as a consequence of the large transverse momentum of the produced jets. A further simplification is that the upper pomeron, although not in its asymptotic regime, is taken with zero total gluon momenta, where the Green function is substantially simpler.

We first discuss the integration on  $R$  (third line in (69)):

$$\begin{aligned} I_1(y) &= \int \frac{d^2 R}{(R - R_1)^2 (R - R_2)^2} \\ &\times G_{s_1/s_0}(R_1, 2(R_2 - R), z_1) \\ &\times G_{s_2/s_0}(R_2, 2(R_1 - R), z_2). \end{aligned} \quad (70)$$

The asymptotics of the two Green functions at large  $s_{1,2}/s_0$  and small  $z_{1,2}$  is easily worked out (see the Appendix). The resulting expression is the same as in (42) with an additional factor 1/2 and a different definition of  $\xi$ :

$$\begin{aligned} G_{s_1/s_0}(R_1, 2(R_2 - R), z_1)_{z_1 \ll 1} &\simeq \frac{e^{\Delta \tilde{y}_1}}{2\pi^{5/2} (a\tilde{y}_1)^{3/2}} \xi_1^{-1} \ln \xi_1 \\ &\times \exp\left(-\frac{\ln^2 \xi_1}{a\tilde{y}_1}\right), \end{aligned} \quad (71)$$

where

$$\xi_1 = \frac{|R - R_1 - R_2| |R + R_1 - R_2|}{2 |R - R_2| z_1}, \quad \tilde{y}_1 = y_1 - y.$$

The same asymptotic expression holds for  $G_{s_2/s_0}$ , with  $1 \leftrightarrow 2$ .

In the region  $y_{1,2} \gg y \gg 1$  one may neglect the dependence on  $y$  in all smoothly varying factors and one may substitute  $\tilde{y}_{1,2} \rightarrow y_{1,2}$  everywhere, with the exception of the exponent. By shifting the integration variable  $R \rightarrow R - R_1 - R_2$  one finds

$$\begin{aligned} I_1(y) &= e^{-2\Delta y} e^{\Delta(y_1+y_2)} \frac{z_1 z_2}{\pi^5 a^3 (y_1 y_2)^{3/2}} \\ &\times \int \frac{d^2 R}{R^2 |R + R_1| |R + R_2| |R + 2R_1| |R + 2R_2|} \\ &\ln \xi_1 \ln \xi_2 \exp\left(-\frac{\ln^2 \xi_1}{ay_1} - \frac{\ln^2 \xi_2}{ay_2}\right), \end{aligned} \quad (72)$$

where

$$\xi_1 = \frac{R |R + 2R_1|}{2z_1 |R + R_1|}, \quad \xi_2 = \xi_1(1 \rightarrow 2). \quad (73)$$

The integral (72) is dominated by the small  $R$  region, so that  $\log(1/R) \sim y_{1,2}^{1/2}$ . One then is allowed to put  $R = 0$  in all factors which are finite in the  $R = 0$  limit and (72) is simplified to

$$\begin{aligned} I_1(y) &= e^{-2\Delta y} e^{\Delta(y_1+y_2)} \frac{1}{4\pi^5 a^3 (y_1 y_2)^{3/2}} \frac{z_1 z_2}{R_1^2 R_2^2} \\ &\int \frac{d^2 R}{R^2} \ln(R/z_1) \ln(R/z_2) \\ &\times \exp\left(-\frac{\ln^2(R/z_1)}{ay_1} - \frac{\ln^2(R/z_2)}{ay_2}\right). \end{aligned} \quad (74)$$

The integration over  $R$  now can be done explicitly (see Appendix) with the result

$$\begin{aligned} I_1(y) &= e^{-2\Delta y} e^{\Delta(y_1+y_2)} \frac{\sqrt{\pi}}{4\pi^4} \left(\frac{1}{a(y_1 + y_2)}\right)^{3/2} \frac{z_1 z_2}{R_1^2 R_2^2} \\ &\left(1 - \frac{2}{a(y_1 + y_2)} \ln^2 \frac{z_1}{z_2}\right) \\ &\times \exp\left(-\frac{\ln^2(z_1/z_2)}{a(y_1 + y_2)}\right). \end{aligned} \quad (75)$$

At large  $y_1$  and  $y_2$ , with  $y_1 = \mathcal{O}(y_2)$ , the second term is subdominant and it may be dropped. The expression is therefore reduced to

$$I_1(y) = e^{-2\Delta y} e^{\Delta(y_1+y_2)} \frac{\sqrt{\pi}}{4\pi^4} \left( \frac{1}{a(y_1+y_2)} \right)^{3/2} \frac{z_1 z_2}{R_1^2 R_2^2} \times \exp\left(-\frac{\ln^2(z_1/z_2)}{a(y_1+y_2)}\right). \quad (76)$$

The next step to evaluate (69) is to integrate the forward Green function, appearing in the first line, over its rapidity variable with the factor arising from  $I_1$ :

$$I_2 = \int_0^\infty dy e^{-2\Delta y} \tilde{G}_{s_0, Q_{13}=0}(r', 2(R_1 - R_2)). \quad (77)$$

Here it has been explicitly indicated that the Green function has to be taken at total momentum of the gluons equal to zero. One then obtains [12]

$$\tilde{G}_s(0, r', r) = \frac{rr'}{2\pi^2} \int_{-\infty}^\infty d\nu s^{-E(\nu)} (r/r')^{2i\nu}, \quad (78)$$

where  $E(\nu)$  is given by (40). By putting this expression into (77) and by integrating over  $y$  one is left with an integral over  $\nu$

$$I_2 = \frac{rr'}{2\pi^2} \int d\nu (r/r')^{2i\nu} \frac{1}{2\Delta + E(\nu)}, \quad (79)$$

where  $r = 2(R_1 - R_2)$ . The distance  $r'$  is of the order of the projectile dimension and it is small, so that the ratio  $r/r'$  for fixed  $R_1 - R_2$  is a large number. The integral (79) can be therefore evaluated by taking the residues of the integrand at the zeros of  $2\Delta + E(\nu)$  in the upper half-plane.

The zeros are located at the points  $\nu = ix_k$ ,  $x_1 < x_2, < \dots$ . The first three points are (see [15])

$$x_1 = 0.3169, \quad x_2 = 1.3718, \quad x_3 = 2.3867. \quad (80)$$

One obtains

$$I_2 = \frac{1}{\alpha_s N} rr' \sum_k c_k (r/r')^{-2x_k}, \quad (81)$$

where

$$c_k^{-1} = \psi'(1/2 - x_k) - \psi'(1/2 + x_k). \quad (82)$$

At large values of  $r/r'$  the nearest pole contributes, so that one finds

$$I_2 \simeq \frac{1}{\alpha_s N} c_1 rr' (r/r')^{-2x_1}. \quad (83)$$

The final average, namely the integration on  $r'$  with the color density of the projectile, gives therefore  $\langle r^{1.64} \rangle$ , to be compared with  $\langle r^2 \rangle$  which is the result obtained for the double parton density when coupling the two pomerons directly to the  $q\bar{q}$  pair.

Collecting all the factors the double parton density corresponding to the triple pomeron picture is finally written as

$$F(x_1, R_1, z_1 | x_2, R_2, z_2) = \frac{c_1}{4^{1+x_1} \pi^{7/2}} \alpha_s^2 \langle r^{1+2x_1} \rangle e^{\Delta(y_1+y_2)} \left( \frac{1}{a(y_1+y_2)} \right)^{3/2} \times \frac{z_1 z_2}{R_1^2 R_2^2 |R_1 - R_2|^{1+2x_1}} \exp\left(-\frac{\ln^2(z_1/z_2)}{a(y_1+y_2)}\right). \quad (84)$$

Analogously to the case of direct coupling of the pomerons to the  $q\bar{q}$  pair, the double parton density contains the factor  $1/R_1^2 R_2^2$ . However, in the triple pomeron case the factor  $1/|R_1 - R_2|^{1+2x_1}$  induces an additional strong positive correlation in transverse space between the two partons.

## 4 Conclusions

In the present paper we have evaluated the double parton distributions, in the case of interactions between very virtual  $q\bar{q}$  pairs in the BFKL regime. After factorizing the hard interactions, the double parton distributions depend on the structure of the blob attached to the projectile (or to the target). Two limiting possibilities have therefore been considered:

- (a) the whole interaction is represented by the exchange of two BFKL pomerons attached directly to the  $q\bar{q}$  pair;
- (b) the process is described by the triple pomeron interaction, where the blob is itself represented by a BFKL pomeron. The two-body correlations in the double distribution are different in the two cases. In the first case the only correlation in the double parton distribution is induced by the configuration taken by the  $q\bar{q}$  pair in transverse space, namely: at a given  $q\bar{q}$  configuration the resulting many-body parton distribution is just a poissonian. The second case is, on the contrary, characterized by a strong correlation in transverse space, as the common source of the two partons is a BFKL ladder rather than the  $q\bar{q}$  pair. The two possibilities are linked, through the AGK rules, to the diffractive events and to the events with rapidity gaps. The relative importance of the two contributions can therefore be inferred by comparing the rates of the two processes.

*Acknowledgements.* M.A. Braun thanks the INFN and the University of Trieste for their financial help during his stay at Trieste. This work was partially supported by the Ministero dell'Università e della Ricerca Scientifica (MURST) through the funds COFIN99.

## Appendix A

In this appendix we shall give some technical details concerning the asymptotics (79) and the calculation of the integral in (72).

First the asymptotics of  $G_s(R, r, r')$  at small  $r'$ . From (37) one concludes that at  $s \rightarrow \infty$  small values of  $\nu$  contribute. So one is lead to study the leading behavior of the integral over  $r_0$  at small  $\nu$  when  $r' \rightarrow 0$ . In this limit the second factor in (37) is singular at  $r_0 = 0$ . So small values of  $r_0$  give the dominant contribution and one can take the first factor out of the integral over  $r_0$  at  $r_0 = 0$ . The integral over  $r_0$  then simplifies to

$$I = \int \frac{d^2 r_0}{(r_0 | r' - r |)^{1-2i\nu}}. \quad (85)$$

(To keep the integral convergent one should take  $\text{Im}\nu > 0$ .) This integral is trivially calculated by going over to the momentum space: if

$$r^{-1+2i\nu} = \int \frac{d^2 q}{(2\pi)^2} e^{iqr} f(q), \quad (86)$$

then

$$I = \int \frac{d^2 q}{(2\pi)^2} e^{iqr'} f^2(q). \quad (87)$$

The function  $f(q)$  is trivially found by the inverse Fourier transformation:

$$f(q) = \pi 2^{1+2i\nu} q^{-1-2i\nu} \frac{\Gamma(1/2 + i\nu)}{\Gamma(1/2 - i\nu)}. \quad (88)$$

Putting this into (83) and doing the integration we find

$$I = (i\pi/2) r'^{4i\nu} \frac{1}{\nu}. \quad (89)$$

Inserting this expression into the integral over  $\nu$  together with all accompanying factors and taking its asymptotics at  $s \rightarrow \infty$  one obtains the desired asymptotics (79).

As to the integral which appears in (72), in the variable  $\beta = \ln R$  it reduces to an integral of the Gaussian type

$$\int_{-\infty}^{+\infty} d\beta (\beta - \beta_1)(\beta - \beta_2) \times \exp\left(-\frac{(\beta - \beta_1)^2}{ay_1} - \frac{(\beta - \beta_2)^2}{ay_2}\right), \quad (90)$$

which can be done in a straightforward manner, with the result (73).

## References

1. C. Goebel, F. Halzen, D.M. Scott, Phys. Rev. D **22**, 2789 (1980); N. Paver, D. Treleani, Nuovo Cimento A **70**, 215 (1982); B. Humpert, Phys. Lett. B **131**, 461 (1983); M. Mekhfi, Phys. Rev. D **32**, 2371 (1985), *ibid.* D **32**, 2380 (1985); T. Sjostrand, M. Van Zijl, Phys. Rev. D **36**, 2019 (1987)
2. T. Akesson et al. (AFS Collaboration), Z. Phys. C **34**, 163 (1987); J. Alitti et al. (UA2 Collaboration), Phys. Lett. B **268**, 145 (1991); F. Abe et al. (CDF Collaboration), Phys. Rev. D **47**, 4857 (1993)
3. F. Abe et al. (CDF Collaboration), Phys. Rev. Lett. **79**, 584 (1997); Phys. Rev. D **56**, 3811 (1997)
4. G. Calucci, D. Treleani, Phys. Rev. D **57**, 503 (1998)
5. Ll. Ametller, D. Treleani, Int. J. Mod. Phys. A **3**, 521 (1988); N. Brown, Mod. Phys. Lett. A **4**, 2447 (1989)
6. G. Pancheri, Y. Srivastava, Phys. Lett. B **182**, 199 (1986); S. Lomatch, F.I. Olness, J.C. Collins, Nucl. Phys. B **317**, 617 (1989)
7. V. Abramovskii, V.N. Gribov, O.V. Kancheli, Yad. Fiz. **18**, 595 (1973) [*Sov. J. Nucl. Phys.* **18**, 308 (1974)]
8. M.A. Braun, D. Treleani, Eur. Phys. J. C **4**, 685 (1998)
9. J. Bartels, V. Del Duca, M. Wusthoff, Z. Phys. C **76**, 75 (1997); V.T. Kim, G.B. Pivovarov, Phys. Rev. D **57**, 1341 (1998)
10. J.R. Forshaw, D.A. Ross, Quantum Chromodynamics and the Pomeron (Cambridge University Press 1997)
11. M.A. Braun, Z. Physik C **71**, 601 (1996)
12. L.N. Lipatov, in Perturbative QCD, edited by A.H. Mueller (World Scientific, Singapore 1989)
13. G. Calucci, D. Treleani, Phys. Rev. D **60**, 054023 (1999)
14. A. Mueller, B. Patel, Nucl. Phys. B **425**, 471 (1994); J. Bartels, M. Wuesthoff, Z. Phys. C **66**, 157 (1995); M.A. Braun, G.P. Vacca, Eur. Phys. J. C **6**, 147 (1999)
15. M.A. Braun, Phys. Lett. B **425**, 354 (1998); Eur. Phys. J. C **6**, 343 (1999)